Advanced Placement Calculus AP Summer Packet

Welcome to Advanced Placement Calculus AB. Before you begin this fall, you need to review some of your Algebra 2 and Pre-Calculus skills. The goal of this packet to review material you learned in previous math courses. These skills are foundational to your success in AP Calculus. Therefore, we suggest that you take some time to complete the following problems. If you are struggling, please feel free to use online resources to help you.

This packet is due the first day of school. It will be graded for points. Each problem should be completed in the space provided. All work must be shown neatly for you to receive full credit. We suggest that you do not wait until the last minute as this assignment will take time. Remember, this assignment is your first introduction of your work to us.

If you have any questions regarding this packet, please reach out to Mrs. Castor at jcastor@wsdweb.org or Dr. Woodland-Smith at lwoodlandsmith@wsdweb.org. Indicate that it is a summer packet question in the subject heading. Please keep in mind that we will be away for portions of the summer and may not get back to you immediately.

Have a good summer and see you in the fall.

The AP Calculus Teachers

Topic 1: Fractional & Negative Exponents

Simplify using only positive exponents.

1) $-3x^{-3}$

2) $-5\left(\dfrac{3}{2}\right)(4-9x)^{-\frac{1}{2}}(-9)$

3) $2\left(\dfrac{2}{2-x}\right)\left(\dfrac{-2}{(2-x)^2}\right)$

4) $\left(16x^2 y\right)^{\frac{3}{4}}$

5) $-\dfrac{x^{-\frac{1}{2}}}{2}\sin\sqrt{x}$

6) $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

7) $-4\left(\dfrac{2x-1}{2x+1}\right)^{-3}\left(\dfrac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right)$

8) $\dfrac{\dfrac{1}{2}(2x+5)^{-\frac{3}{2}}}{\dfrac{3}{2}}$

9) $\left(\dfrac{1}{x^{-2}}+\dfrac{4}{x^{-1}y^{-1}}+\dfrac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

Topic 2: Domain

Find domain of the following functions:

1) $y = \dfrac{3x - 2}{4x + 1}$

2) $y = \dfrac{x^2 - 4}{2x + 4}$

3) $y = \dfrac{x^2 - 5x - 6}{x^2 - 3x - 18}$

4) $y = \dfrac{2^{2-x}}{x}$

5) $y = \sqrt{x - 3} - \sqrt{x + 3}$

6) $y = \dfrac{\sqrt{2x - 9}}{2x + 9}$

7) $y = \dfrac{x^2 + 8x + 12}{\sqrt[4]{x + 5}}$

8) $y = \sqrt{x^2 - 5x - 14}$

9) $y = \dfrac{\sqrt[3]{x - 6}}{\sqrt{x^2 - x - 30}}$

10) $y = \log(2x - 12)$

11) $y = \sqrt{\tan x}$

12) $y = \dfrac{x}{\cos x}$

Topic 4: Solving Inequalities (quadratic)

Write the following absolute value expression as piecewise expressions

1) $\left| x^2 - 1 \right|$

2) $\left| x^2 + x - 12 \right|$

3) $\left| x^2 + 4x + 4 \right|$

Solve the following by factoring and making appropriate sign charts.

4) $x^2 - 16 > 0$

5) $x^2 + 6x - 16 > 0$

6) $x^2 - 3x \geq 10$

7) $2x^2 + 4x \leq 3$

8) $x^3 + 4x^2 - x \geq 4$

9) $2\sin^2 x \geq \sin x \quad 0 \leq x \leq 2\pi$

Topic 5: Special Factorization

Factor completely

1) $x^3 + 8$

2) $x^3 - 8$

3) $27x^3 - 125y^3$

4) $x^4 + 11x^2 - 80$

5) $ac + cd - ab - bd$

6) $2x^2 + 50y^2 - 20xy$

7) $x^2 + 12x + 36 - 9y^2$

8) $x^3 - xy^2 + x^2y - y^3$

9) $(x-3)^2 (2x+1)^3 + (x-3)^3 (2x+1)^2$

Topic 6: Function Transformation

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:
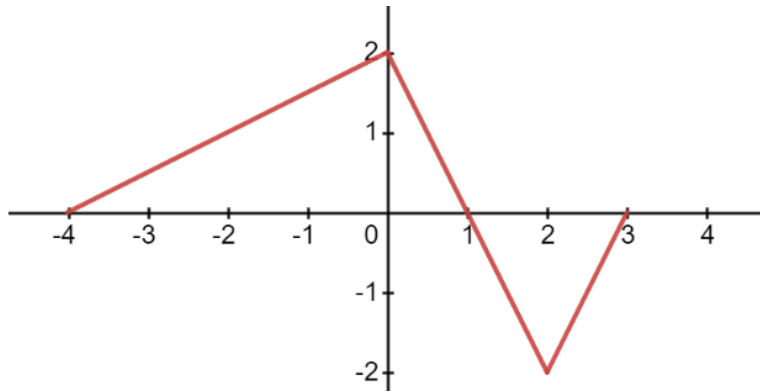
1) $f(x) - 4$

2) $f(x-4)$
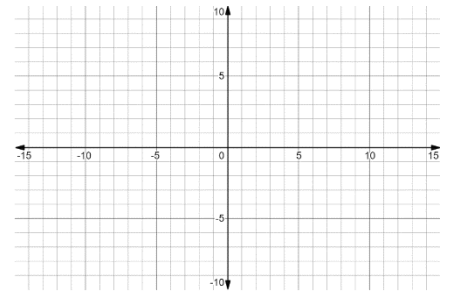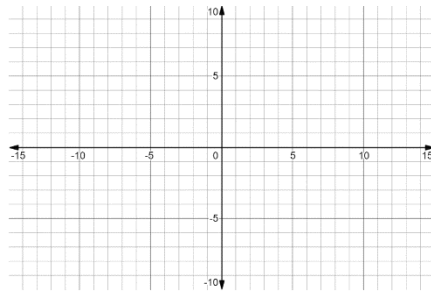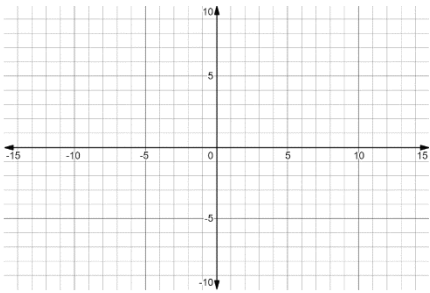
3) $-f(x+2)$

4) $5f(x) + 3$

5) $f(2x)$

6) $|f(x)|$

Here is a graph of $y = f(x)$. Sketch the following graphs
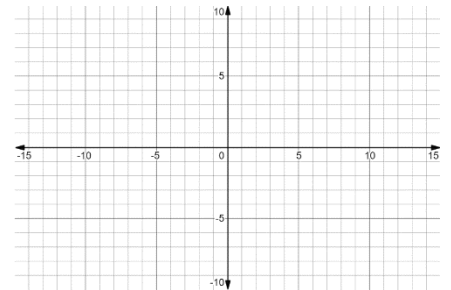

7) $y = 2f(x)$

8) $y = -f(x)$
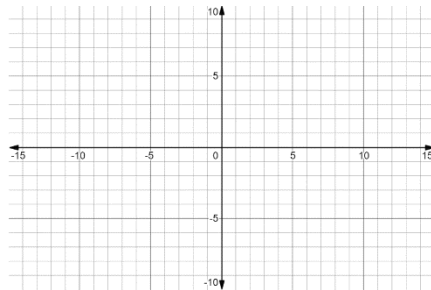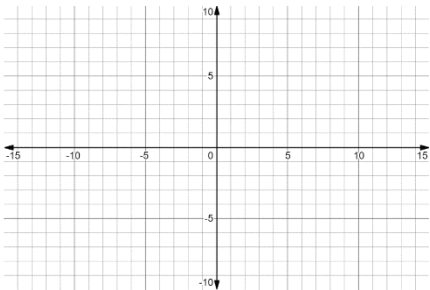
9) $y = f(x-1)$

10) $y = f(x+2)$

11) $y = |f(x)|$

12) $y = f|x|$

Topic 7: Factor Theorem (*p* over *q* method/synthetic division)

Use the *p* over *q* method and synthetic division to factor the polynomial $P(x)$. Then solve $P(x) = 0$.

1) $P(x) = x^3 + 4x^2 + x - 6$

2) $P(x) = x^3 + 5x^2 - 2x - 24$

3) $P(x) = x^3 - 6x^2 + 3x - 10$

4) $P(x) = x^3 + 2x^2 - 19x - 20$

5) $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$

6) $P(x) = x^4 + 11x^3 + 41x^2 + 61x + 30$

Topic 8: Even and Odd Functions

Show work to determine if the relation is even, odd, or neither

1) $f(x) = 2x^2 - 7$

2) $f(x) = -4x^3 - 2x$

3) $f(x) = 4x^2 - 4x + 4$

4) $f(x) = x = \dfrac{1}{x}$

5) $f|x| = |x| - x^2 + 1$

6) $5x^2 - 6y = 1$

7) $y = e^x - \dfrac{1}{e^x}$

8) $3y^3 = 4x^3 + 1$

9) $3x = |y|$

Topic 9: Solving Quadratic Equations and Quadratic Formula

Solve each equation

1) $7x^2 - 3x = 0$

2) $4x(x-2) - 5x(x-1) = 2$

3) $x^3 + 6x + 4 = 0$

4) $2x^2 - 3x + 3 = 0$

5) $2x^2 - (x+2)(x-3) = 12$

6) $x + \dfrac{1}{x} = \dfrac{13}{6}$

7) $x^4 - 9x^2 + 8 = 0$

8) $x - 10\sqrt{x} + 9 = 0$

9) $\dfrac{1}{x^2} - \dfrac{1}{x} = 6$

Topic 10: Asymptotes

For each function, find the equation of both the vertical asymptote(s) and horizontal asymptotes (if they exist)

1) $y = \dfrac{x}{x-3}$

2) $y = \dfrac{x+4}{x^2-1}$

3) $y = \dfrac{x+4}{x^2+1}$

4) $y = \dfrac{x^2-2x+1}{x^2-3x-4}$

5) $y = \dfrac{x^2-9}{x^3+3x^2-18x}$

6) $y = \dfrac{2x^2+6x}{x^3-3x^2-4x}$

7) $y = \dfrac{x^2-x-6}{x^3-x^2+x-6}$

8) $y = \dfrac{2x^3}{x^3-1}$

9) $y = \dfrac{\sqrt{x}}{2x^2-10}$

Topic 11: Complex Fractions

Simplify the following

1) $\dfrac{x}{x - \dfrac{1}{2}}$

2) $\dfrac{\dfrac{1}{x} + 4}{\dfrac{1}{x} - 2}$

3) $\dfrac{x - \dfrac{1}{x}}{x + \dfrac{1}{x}}$

4) $\dfrac{\dfrac{3}{x} - \dfrac{4}{y}}{\dfrac{4}{x} - \dfrac{3}{y}}$

5) $\dfrac{1 - \dfrac{2}{3x}}{x - \dfrac{4}{9x}}$

6) $\dfrac{\dfrac{x^2 - y^2}{xy}}{\dfrac{x + y}{y}}$

7) $\dfrac{x^{-3} - x}{x^{-2} - 1}$

8) $\dfrac{\dfrac{x}{1 - x} + \dfrac{1 + x}{x}}{\dfrac{1 - x}{x} + \dfrac{x}{1 + x}}$

9) $\dfrac{\dfrac{4}{x - 5} + \dfrac{2}{x + 2}}{\dfrac{2x}{x^2 - 3x - 10} + 3}$

Topic 12: Composition of Functions

If $f(x) = x^2$, $g(x) = 2x - 1$, and $h(x) = 2^x$, find the following

1) $f(g(2))$

2) $g(f(2))$

3) $f(h(-1))$

4) $h(f(-1))$

5) $g\left(f\left(h\left(\dfrac{1}{2}\right)\right)\right)$

6) $f(g(x))$

7) $g(f(x))$

8) $g(g(x))$

9) $f(h(x))$

Use the table of values to evaluate each expression.

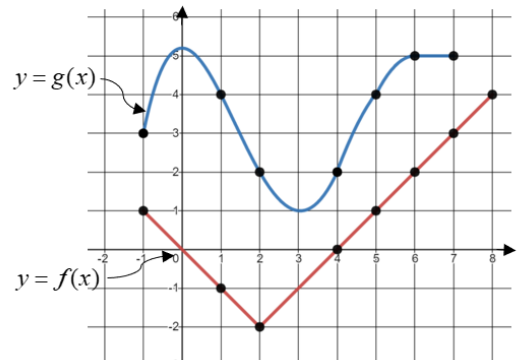| x	-3	-2	-1	0	1	2	3
$f(x)$	11	9	7	5	3	1	-1
$g(x)$	-8	-3	0	1	0	-3	-8

10) $f(g(1))$

11) $g(f(2))$

12) $f(f(3))$

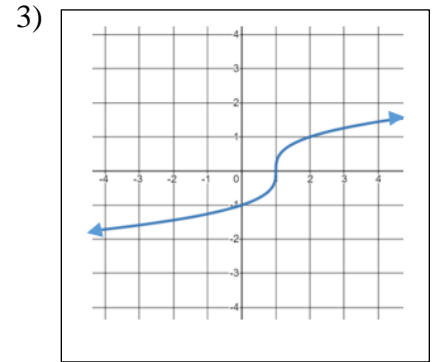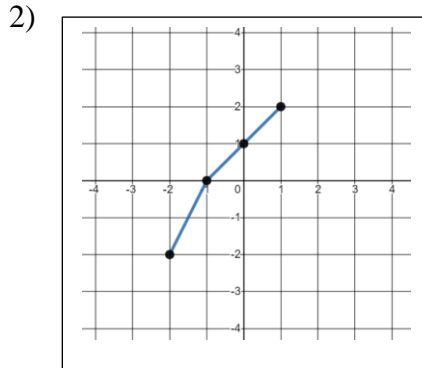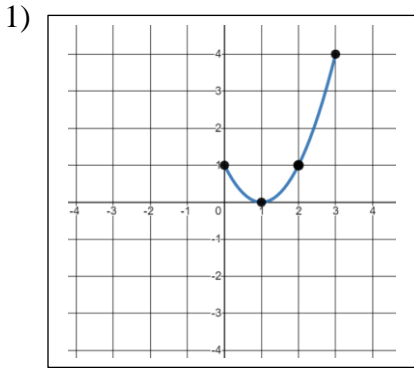Evaluate each expression using the graphs of $y = f(x)$ and $y = g(x)$

13) $g(f(-1))$

14) $f(g(4))$

15) $g(g(2))$

Topic 13: Inverse Functions

Draw the graph of the inverse for each of the graphs below.

1)

2)

3)

Verify the functions f and g are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$.

4) $f(x) = 3x + 4;\quad g(x) = \dfrac{1}{3}(x-4)$

5) $f(x) = x^3 - 8;\quad g(x) = \sqrt[3]{x+8}$

6) $f(x) = \dfrac{1}{x};\quad g(x) = \dfrac{1}{x}$

7) $f(x) = \dfrac{2x+3}{x+4};\quad g(x) = \dfrac{4x-3}{2-x}$

Find the inverse for each of the following. State the domain and range of f and f^{-1}.

8) $f(x) = 4x + 2$

9) $f(x) = \dfrac{2}{3+x}$

10) $f(x) = x^2 + 4;\ x \geq 0$

Topic 14: Solving Rational (Fractional) Equations

Solve each equation for x

1) $\dfrac{2}{3} - \dfrac{5}{6} = \dfrac{1}{x}$

2) $x + \dfrac{6}{x} = 5$

3) $\dfrac{x+1}{3} - \dfrac{x-1}{2} = 1$

4) $-\dfrac{x-5}{x+1} = \dfrac{3}{5}$

5) $\dfrac{60}{x} - \dfrac{60}{x-5} = \dfrac{2}{x}$

6) $\dfrac{2}{x+5} + \dfrac{1}{x-5} = \dfrac{16}{x^2 - 25}$

7) $\dfrac{x}{x-2} + \dfrac{2x}{4-x^2} = \dfrac{5}{x+2}$

8) $\dfrac{x}{2x-6} - \dfrac{3}{x^2 - 6x + 9} = \dfrac{x-2}{3x-9}$

9) $\dfrac{2x+3}{x-1} = \dfrac{10}{x^2 - 1} + \dfrac{2x-3}{x+1}$

Topic 15: Concepts in Trigonometry

Solve the following problems.

If point P is on the terminal side of θ, find all 6 trig functions of θ. Draw a picture

1) $P(-2,4)$

2) $P(\sqrt{5},-2)$

3) If $\cos\theta = \dfrac{5}{13}$, θ in quadraant II,
find $\sin\theta$ and $\tan\theta$

4) If $\cot\theta = 3$, θ in quadrant III,
find $\sin\theta$ and $\cos\theta$

Find the exact value of the following without a calculator:

5) $\sin^2 225° - \cos^2 300°$

6) $\left(6\sec180° - 4\cot90°\right)^2$

7) $\left(4\cos30° - 6\sin120°\right)^{-2}$

Solve the following triangles (3 decimal place accuracy)

$A =$	$a = 21.7$	
8) $B = 16°$	$b =$	
$C = 90°$	$c =$	

$A =$	$a = 6$ feet	
9) $B =$	$b =$	
$C = 90°$	$c = 95$ inches	

Topic 16: Solving Trigonometric Equations

Solve each equation on the interval $[0, 2\pi)$

1) $\sin x = \dfrac{1}{2}$

2) $\cos^2 x = \cos x$

3) $2\cos x + \sqrt{3} = 0$

4) $4\sin^2 x = 1$

5) $2\sin^2 x + \sin x = 1$

6) $\cos^2 x + 2\cos x = 3$

7) $2\sin x \cos x + \sin x = 0$

8) $8\cos^2 x - 2\cos x = 1$

9) $\sin^2 x - \cos^2 x = 0$